Indian Statistical Institute, Bangalore
B. Math (Hons.) Third Year
Second Semester - Differential Geometry II

Back Paper Exam — Date: June 15, 2018
Maximum Marks: 45 — Duration: 3 hours

1. Show that the space of orthogonal matrices $O(n)$ is a manifold and find the dimension of $O(n)$. [3 + 2 = 5]

2. (a) Show that there exists no submersion $f : \mathbb{S}^n \to \mathbb{R}^m (m, n \geq 1)$.
   (b) Show that every open topological subgroup is a closed subgroup. [4 + 4 = 8]

3. Define a p-form on a smooth manifold. Verify that the smoothness of a p-form is independent of local parameterizations. [2 + 3 = 5]

4. Show that there can be atmost two orientations on a connected manifold. [5]

5. Equip the unitary group $U(m)$ with the Riemannian metric g given by $g(Z, W) = Re(\text{ trace } (\overline{Z}^t \cdot W))$. Show that for each $p \in U(m)$, the left translation $L_p : U(m) \to U(m)$ is an isometry. [7]

6. Determine which of the following vector fields are complete.
   (a) $X(x_1, x_2) = (x_1, x_2, 1, 0), (x_1, x_2) \in \mathbb{R}^2$
   (b) $X(x_1, x_2) = (x_1, x_2, 1, 0), (x_1, x_2) \in \mathbb{R}^2 \setminus \{(0, 0)\}$
   (c) $X(x_1, x_2) = (x_1, x_2, -x_2, x_1), (x_1, x_2) \in \mathbb{R}^2 \setminus \{(0, 0)\}$
   (d) $X(x_1, x_2) = (x_1, x_2, 1 + x_1^2, 0), (x_1, x_2) \in \mathbb{R}^2$
   (e) $X(x_1, x_2) = (x_1, x_2, x_2, x_1), (x_1, x_2) \in \mathbb{R}^2$ [15]